Journal Club

Quantifying Lottery Choice Complexity

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Introduction

Paper

Enke, B., & Shubatt, C. (2023). **Quantifying Lottery Choice Complexity**. *SSRN Electronic Journal.* https://doi.org/10.2139/ssrn.4568763

1. Research Question
2. Conceptual Framework
3. Experimental Design
4. Complexity Indices
5. Results
6. Discussion

Research Question

Can we find quantitative and interpretable metrics of objective and subjective complexity of lottery choice problems with prediction power on human performance?

- Proposed metrics for complexity
- Model the effect of complexity on human performance
- Construction and test of potential complexity indices
- Experiment on EV tasks and choice tasks
- Parameter/effect estimation for the indices
- Analysis on the predictiveness of the model

Conceptual FW I: Objective Complexity

Revealed Objective Complexity (ROC) ⟶ empirical

- **Error-based** - error rate in decision-making task with lottery choice
- $ROC_{A,B} = P(d_c \notin \underset{x_A,x_B}{\operatorname{argmax}} EU(x)) \longrightarrow$ choice task: choose b/w lotteries, **BUT utility function unknown**
- $ROC_{A,B}^{EV} = P(d_c \notin \underset{x_A,x_B}{\operatorname{argmax}} EV(x)) \longrightarrow$ EV task: select the lottery with highest EV, instead
- **Assumption 1**: errors arise mainly due to complexity of aggregating probabilities and outcomes ("aggregation complexity")
- **Assumption 2**: errors in EV tasks is predictive of errors in choice tasks $\longrightarrow ROC_{A,B} = f\left(ROC_{A,B}^{EV}\right)$ for some monotonically increasing f

Objective Problem Complexity (OPC) ⟶ estimated

- **Predicted error rates** with estimated parameters and choice features
- $OPC_{C,D} \coloneqq \sum_{i=0}^{N} \hat{\beta}_i f_i^{C,D} + \sum_{j=1}^{2} |EV(C) - EV(D)|^j$
  - Aggregation complexity: represented by a finite set of choice features, each with a parameter
  - Proximity of aggregated/expected values
- Parameter estimation: regression of observed error rates from a sample of EV tasks
  - $ROC_{A,B}^{EV} = \sum_{i=0}^{N} \beta_i f_i^{C,D} + \sum_{j=1}^{2} |EV(C) - EV(D)|^j + \epsilon_{A,B}$

Conceptual FW II: Aggregation Complexity

Implied Inverse Logit Precision ⟶ empirical

- Particularly focused on **errors induced by complexity**
- Logit model: $P(A) = F\big(EU(A) - EU(B)\big) = \frac{1}{1+e^{-\eta[EU(A)-EU(B)]}}$
  - η: precision parameter
  - Capturing aggregation complexity and resulting noise, separately from proximity to indifference
- $s_{A,B} := \frac{1}{\eta_{A,B}} = \frac{EU(B)-EU(A)}{\ln\left(\frac{1}{P(A)}-1\right)}$
- $s_{A,B}^{EV} := \frac{1}{\eta_{A,B}^{EV}} = \frac{EV(B)-EV(A)}{\ln\left(\frac{1}{P(A)}-1\right)}$
- Assumption: $\eta_{A,B} = \eta_0 + \eta_1 \frac{1}{s_{A,B}^{EV}}$, with $\eta_1 > 0$

Objective Aggregation Complexity (OAC) ⟶ estimated

- **Predicted inverse logit precision** with estimated parameters and choice features
- $OAC_{C,D} = \sum_{i=0}^{N} \hat{\alpha}_i f_i^{C,D}$
- Parameter estimation: regression of implied inverse logit precision from a sample of EV tasks
  - $s_{A,B}^{EV} = \frac{EV(B)-EV(A)}{\ln\left(\frac{1}{P(A)}-1\right)} = \sum_{i=0}^{N} \alpha_i f_i^{A,B} + \epsilon_{A,B}$
  - $s_{A,B}^{EV}$ can be calculated since $EV(B) - EV(A)$ and $P(A)$ are observed from sample

Conceptual FW III: Individual Lottery

Objective Lottery Complexity (OLC)

- OPC is on problem/choice level i.e. the complexity of making a choice or selecting the highest EV among lotteries
- OLC is measured similarly as OAC but instead with a lottery and a certain payment
- **Predicted inverse logit precision** with estimated parameters and lottery features
- $OLC_C = \sum_{i=0}^{N} \hat{\xi}_i f_i^C + \epsilon_{C,D}$
  - Only the features of non-degenerate lottery enter the function
  - i.e. only the lottery involving at least two different payoffs with positive probability matters, not the certain payment
- Parameter estimation: regression of implied inverse logit precision from a sample of EV tasks
  - $s_{A,B}^{EV} = \frac{EV(B)-EV(A)}{\ln\left(\frac{1}{P(A)}-1\right)} = \sum_{i=0}^{N} \xi_i f_i^A + \epsilon_{A,B}$
  - $s_{A,B}^{EV}$ can be calculated since $EV(B) - EV(A)$ and $P(A)$ are observed from sample

Conceptual FW IV: Subjective Complexity

Subjective Problem Complexity (SPC)

- $SPC_{C,D} \coloneqq \sum_{i=0}^{N} \hat{\beta}_i f_i^{C,D} + \sum_{j=1}^{2} |EV(C) - EV(D)|^j$
- Parameter estimation: regression of subjective/elicited error rates from a sample of EV tasks
  - $\sum_{i=0}^{N} \beta_i f_i^{C,D} + \sum_{j=1}^{2} |EV(C) - EV(D)|^j + \epsilon_{A,B}$

Subjective Aggregation Complexity (SAC)

- $SAC_{C,D} = \sum_{i=0}^{N} \hat{\alpha}_i f_i^{C,D}$
- Parameter estimation: regression of implied inverse logit precision from a sample of EV tasks
  - $s_{A,B}^{EV} = \frac{EV(B)-EV(A)}{\ln\left(\frac{1}{P(A)}-1\right)} = \sum_{i=0}^{N} \alpha_i f_i^{A,B} + \epsilon_{A,B}$
  - $s_{A,B}^{EV}$ can be calculated since $EV(B) - EV(A)$ is known and average subjective probability $P(A)$ is elicited

Subjective Lottery Complexity (SLC)

- $SLC_C = \sum_{i=0}^{N} \hat{\xi}_i f_i^{C} + \epsilon_{C,D}$
- Parameter estimation: regression of implied inverse logit precision from a sample of EV tasks
  - $s_{A,B}^{EV} = \frac{EV(B)-EV(A)}{\ln\left(\frac{1}{P(A)}-1\right)} = \sum_{i=0}^{N} \xi_i f_i^{A} + \epsilon_{A,B}$
  - $s_{A,B}^{EV}$ can be calculated since $EV(B) - EV(A)$ is known and average subjective probability $P(A)$ is elicited

Experiment Design I: Task (1/2) EV

Which lottery has the highest average payoff if the computer runs it many, many times?
Please select one.

Lottery A	Lottery B
Prob. 50%: Get $30 **Prob. 50%:** **Get $0**	**Prob. 100%:** Get $9

How certain are you that each lottery has the highest average payoff?
Please allocate 100 certainty points.

[ ] points [ ] points

100 points left to allocate.

- Jargons like "expected value" were avoided – wording of instructions and framing of questions
- Each lottery is run 100,000 times, payouts is recorded, and average payout is computed across all runs.
- There is a correct answer to each problem, and if selected, +$10 bonus
- Cognitive Uncertainty elicitation

Experiment Design II: Task (2/2) Choice

Which lottery do you choose?
Please select one.

Lottery A		Lottery B	
Prob. 80%:	Get $42	**Prob. 100%:**	Get $28
Prob. 20%:	Get $4		

How certain are you that you actually prefer the lottery you chose above?

Fully certain I prefer the lottery I didn't choose — Fully certain I prefer the lottery I chose

0% 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%

I am **PLEASE CLICK SLIDER** points left to allocate.

- Binary choice only
- Loss involved and incentivized: receive a budge equal to the largest possible loss for each choice set

Experiment Design III: Setup

Experiment	Description	Problems	Subjects	Decisions
EV Tasks	Indicate lottery with highest EV & *CU* elicitation	2,100 procedurally generated & 120 targeted	1,148	57k
Choice Tasks from PEA	Lottery choice problems	10,423 procedurally generated	11,681	975k
Choice Tasks	Lottery choice problems & *CU* elicitation	500 procedurally generated	250	12.5k

Notes. PEA = Peterson et al. (2021). *CU* = Cognitive uncertainty.

EV tasks – all self-collected on *Prolific*
- Composition
  - 95% binary choice
    - 30% non-degenerate lottery vs certain payment
    - 70% number of states varies from 2 to 7
  - 5% among 3-5 lotteries
- Each instance completed by at least 20 subjects
- Incentive: $6 completion + $10 bonus for ½ subjects

Indices estimation only

Choice tasks – other's dataset on *AMT* and self-collected on *Prolific*
- Peterson *et al.* 2021
  - Quasi-random generation
  - Didn't pay out losses but truncated from below at zero
- Self-collected
  - Realize losses with a pre-assigned budget
  - Elicitation of cognitive certainty
  - Incentive: $3.5 completion + realized bonus for 1/5 subjects

Behavioral response analysis

Complexity Indices I: Generation

Feature	Defined on	Formal definition
Number of states	Option	k_j
Payout range	Option	$\max\{x_1^j, \ldots, x_{k_j}^j\} - \min\{x_1^j, \ldots, x_{k_j}^j\}$
Variance	Option	$\sum_{s=1}^{k_j} p_s^j \left(x_i^j\right)^2 - \left(\sum_{s=1}^{k_j} p_s^j x_s^j\right)^2$
Payout variance	Option	$1/k_j \sum_{s=1}^{k_j} (x_s^j - \bar{x}^j)^2$
Probability variance	Option	$1/k_j \sum_{s=1}^{k_j} (p_s^j - \bar{p}^j)^2$
Magnitude	Option	$1/k_j \sum_{s=1}^{k_j} \lvert x_s^j \rvert$
Pure Gains	Option	$\mathbb{1}\{x_s^j \geq 0 \forall j\}$
Mixed	Option	$\mathbb{1}\{\exists x_s^j > 0 \wedge \exists x_s^j < 0\}$
Pure Loss	Option	$\mathbb{1}\{x_s^j < 0 \forall j\}$
Distance to certainty	Option	$1/k_j \sum_{s=1}^{k_j} \min\{p_s^j; 1 - p_s^j\}$
Payout-weighted dist. to certainty	Option	$1/k_j \sum_{s=1}^{k_j} \lvert x_s^j \rvert \min\{p_s^j; 1 - p_s^j\}$
Entropy	Option	$\sum_{s=1}^{k_j} p_s^j(-ln(p_s^j))$
Normalized payout dispersion	Option	$1/k_j \sum_{s=1}^{k_j} \frac{\lvert x_s^j - \bar{x}^j \rvert}{\lvert \bar{x}^j \rvert}$
Normalized Variance	Option	$(1/\text{Magn.}^2) \cdot \text{Var}$, with Magn., Var. as defined above
Irregular probabilities	Option	$\mathbb{1}\left(p_s^j \notin \{0.01, 0.05, 0.1, \ldots, 0.9, 0.95, 0.99\} \text{ for } s = 1, \ldots, k_j\right)$
CDF self-distance	Option	$\sum_{s=1}^{k_j} \lvert x_s^j - EV(j) \rvert p_s^j$
Compound	Option	
Compound Range	Option	Range of distribution of unknown p
Weak dominance	Choice set	$\mathbb{1}\{F_A(x) \leq F_B(x) \forall x\}$
Excess dissimilarity	Choice set	$\int_{\mathbb{R}} \lvert F_A(x) - F_B(x) \rvert dx - \lvert EV(A) - EV(B) \rvert$
Average absolute payoff difference	Choice set	$1/k \sum_{s=1}^{k} \lvert x_s^A - x_s^B \rvert$
Probability difference	Choice set	$\sum_{x \in X} \lvert f_A(x) - f_B(x) \rvert$, where $X = \{x_1^A, \ldots x_{k_A}^A\} \cup \{x_1^B, \ldots, x_{k_B}\}$

LASSO Regression

- Regression of OPC, SPC, OLC, SLC on all features
- 75%-25% train-test split
- Minimize MSE (mean squared error) in train set
- For option features over a choice set, calculate linear, log, and square averages across all options
- No interaction between features allows

Handcrafted Indices

- Many features are intra-correlated
- Manually selected and created some features that are based on fewer features and represent a broad class of features each
- Handcrafted indices turned out to be almost perfectly correlated with LASSO explanatory indices
- Results are virtually the same when using LASSO-generated indices vs handcrafted indices

Complexity Indices II: Parameter Estimation

	Dependent variable:		
	Observed error rate	Implied obj. logit imprecision s^{EV}	
Index:	*OPC*	*OAC*	*OLC*
	(1)	(2)	(3)
Log excess dissimilarity	0.052*** (0.00)	1.69*** (0.15)	
No dominance	0.068*** (0.01)	0.67* (0.35)	
Average log payout magnitude	0.036*** (0.00)	1.12*** (0.18)	
Average log number of states	0.045*** (0.02)	2.07*** (0.59)	
Frac. lotteries involving loss	0.029*** (0.01)	1.17*** (0.32)	
Frac. lotteries involving compound prob.	0.15*** (0.03)	4.93*** (1.26)	
Absolute EV difference	-0.032*** (0.01)		
Absolute EV difference sqr.	0.0014*** (0.00)		
Log variance			1.06*** (0.11)
Log payout magnitude			0.90*** (0.24)
Log number of states			2.10*** (0.48)
1 if involves loss			0.96*** (0.36)
1 if involves compound prob.			2.14*** (0.74)
Constant	0.027 (0.03)	-4.12*** (0.81)	-5.67*** (0.91)
Observations	1587	1587	935
R^2	0.31	0.20	0.23

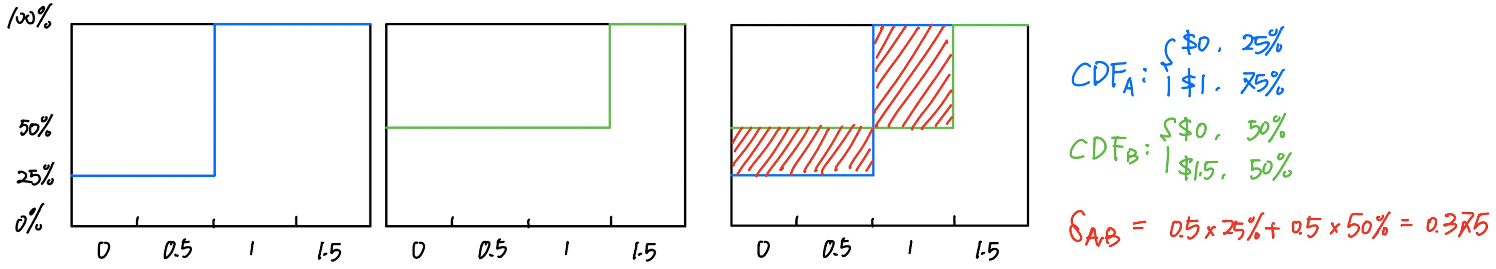
	Dependent variable:		
	Cognitive uncertainty	Implied subj. logit imprecision $\hat{s}$	
Index:	*SPC*	*SAC*	*SLC*
	(1)	(2)	(3)
Log excess dissimilarity	0.018*** (0.00)	1.79*** (0.16)	
No dominance	0.043*** (0.01)	0.83** (0.41)	
Average log payout magnitude	0.0036* (0.00)	1.43*** (0.20)	
Average log number of states	0.052*** (0.01)	3.10*** (0.63)	
Frac. lotteries involving loss	0.028*** (0.00)	2.24*** (0.35)	
Frac. lotteries involving compound prob.	0.12*** (0.01)	6.84*** (1.35)	
Absolute EV difference	-0.0037** (0.00)		
Absolute EV difference sqr.	0.00010 (0.00)		
Log variance			1.17*** (0.13)
Log payout magnitude			1.05*** (0.29)
Log number of states			2.95*** (0.53)
1 if involves loss			1.71*** (0.39)
1 if involves compound prob.			3.14*** (0.84)
Constant	0.021** (0.01)	-5.35*** (0.88)	-6.75*** (1.03)
Observations	1587	1587	935
R^2	0.38	0.24	0.26

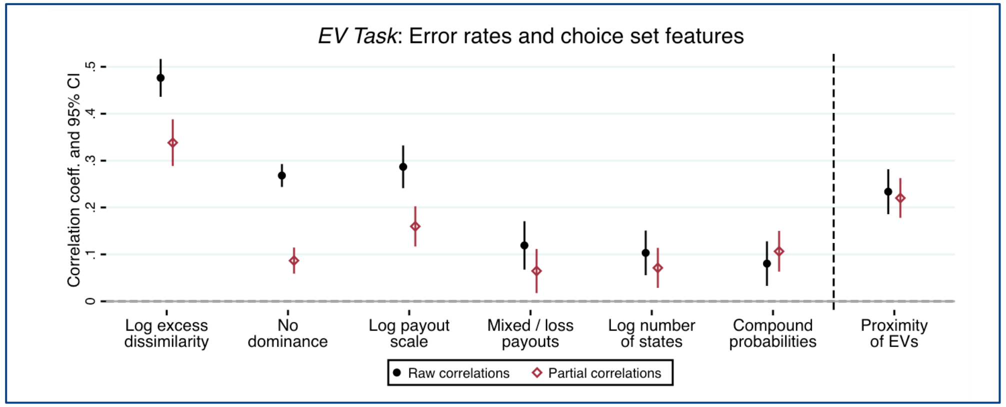
These results are on the handcrafted indices

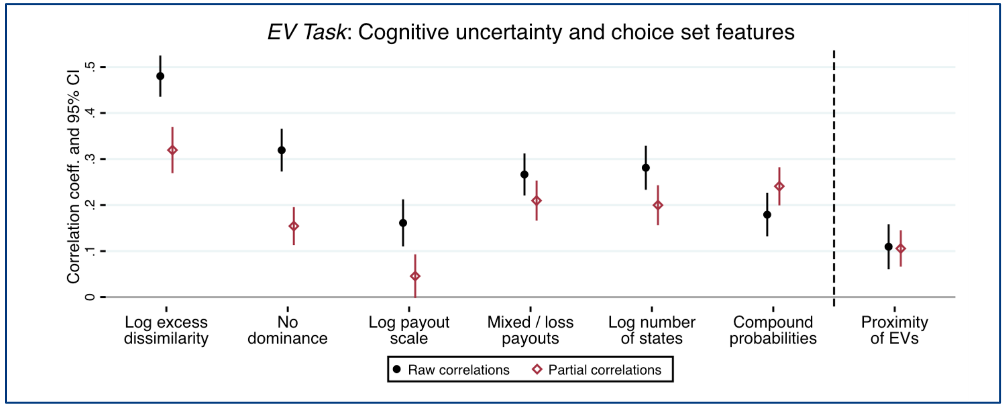
Complexity Indices III: Notable Features

Excess Dissimilarity

- Degree to which lotteries are dissimilar from each other, in addition to difference in expected value
- **Dissimilarity**: overlay the CDF of two lotteries and calculate summed area between the two
  - Wasserstein 1-distance
  - $\delta_{A,B} = \int_{\mathbb{R}} |F_A(x) - F_B(x)|$
- **Excess dissimilarity**:
  - $d_{A,B} = \delta_{A,B} - |EV(A) - EV(B)|$
- Intuition:
  - DM assesses two lotteries by states
  - If for most states their outcomes are similar, then they are considered similar
  - DM only needs to focus on the states that are not similar to assess their preference



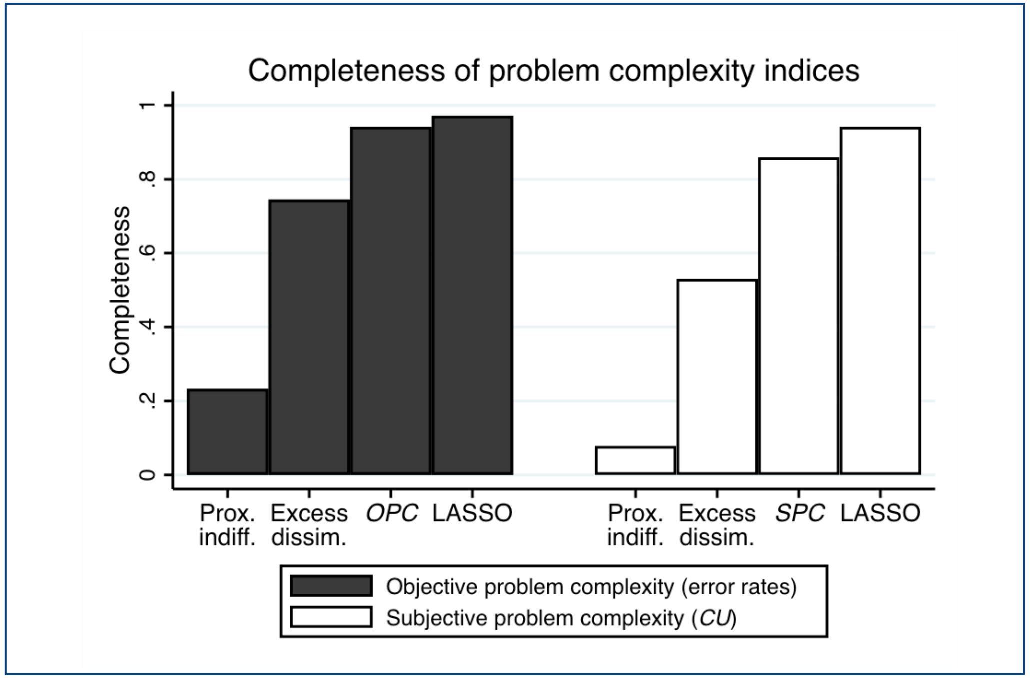




Complexity Indices IV: Completeness

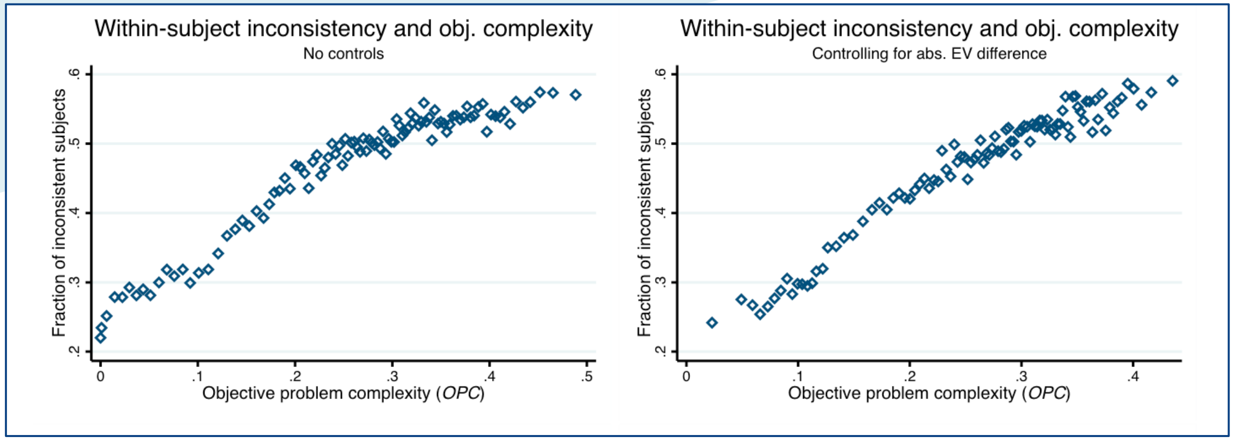
Convolutional Neural Network

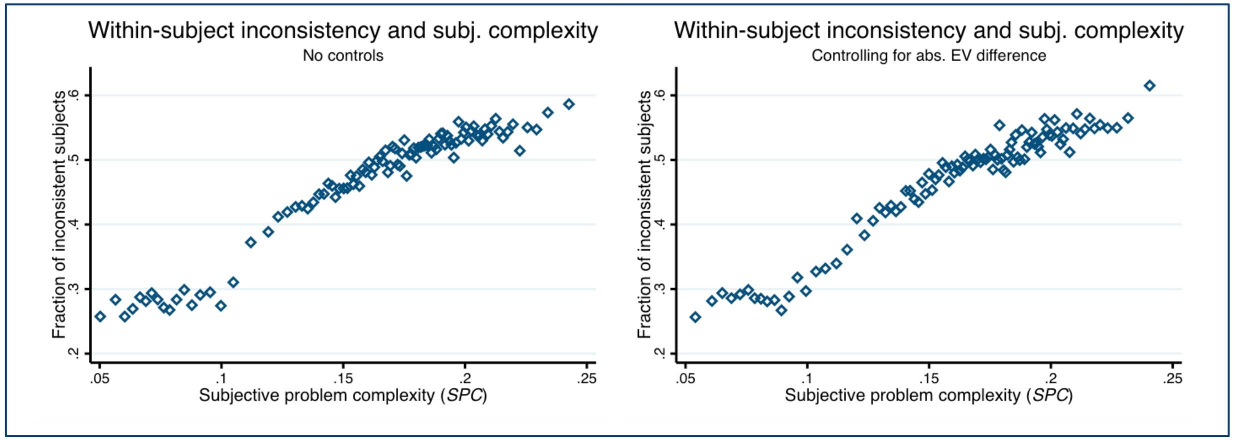
- Input:
  - Raw lottery features: payouts and probabilities
  - All previous features used in LASSO regressions
- Out: error rates (objective), or cognitive uncertainty (subjective)
- **Completeness**: ratio of variance explained of the index and that by the CNN



- 13 handcrafted indices are as good as 46 LASSO indices
- Excess dissimilarity captures most of the complexity
- Proximity to indifference is not enough

Results I: Complexity and Noise

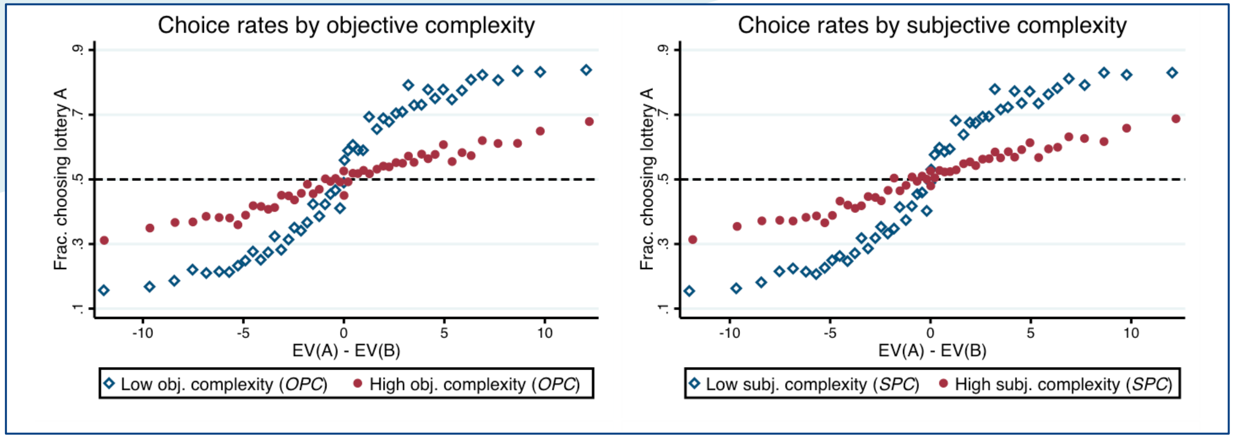


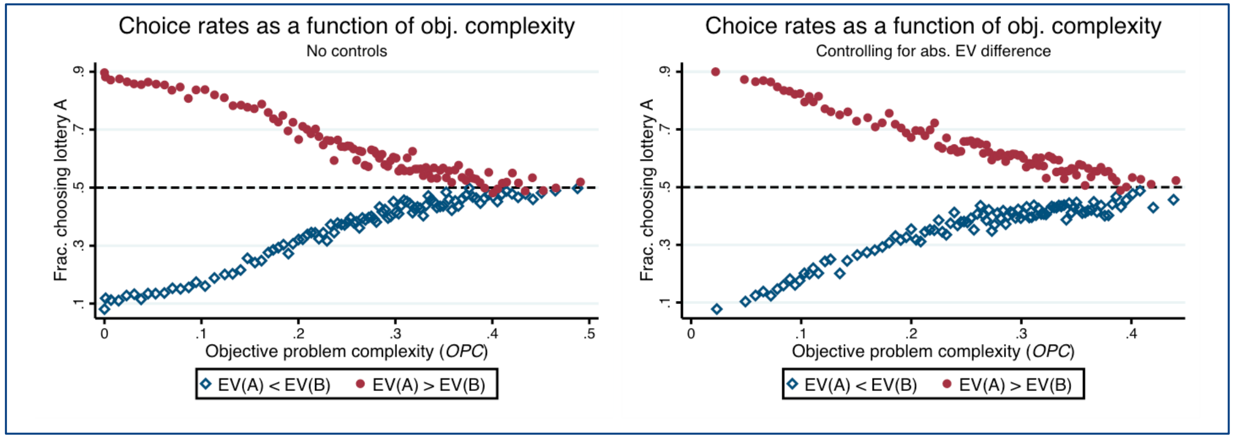


Across-trial inconsistency: within-subject choice inconsistency

- Problem-level, fraction of subjects who made inconsistent choice at least once
- Only on choice problems where the difference between EVs is at least $0.20
- Binned, 104 problems per dot
- No control vs. control for EV difference (i.e. "pure" aggregation complexity)
- OPC 0 -> 0.5, ATI 35% ↑

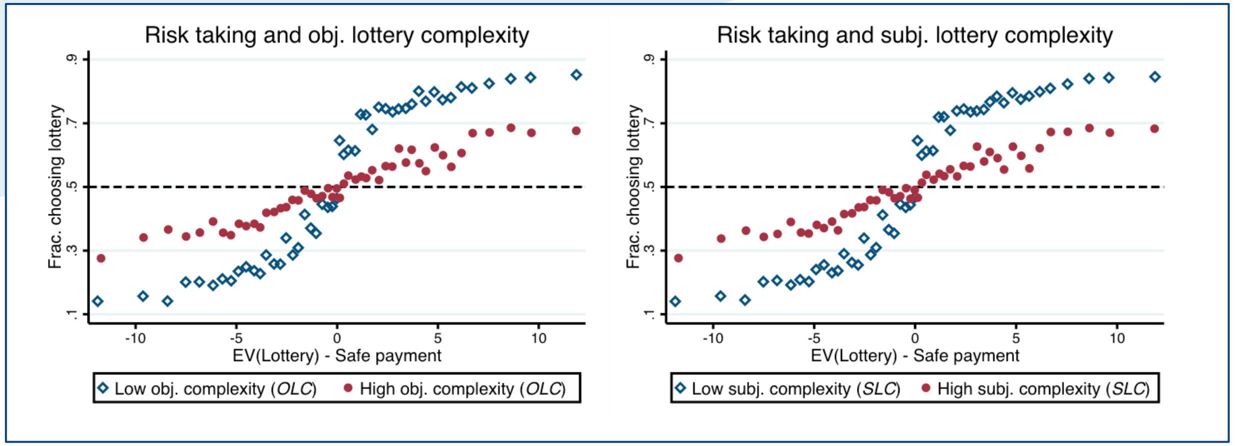
Results II: Compression of Choice Rate



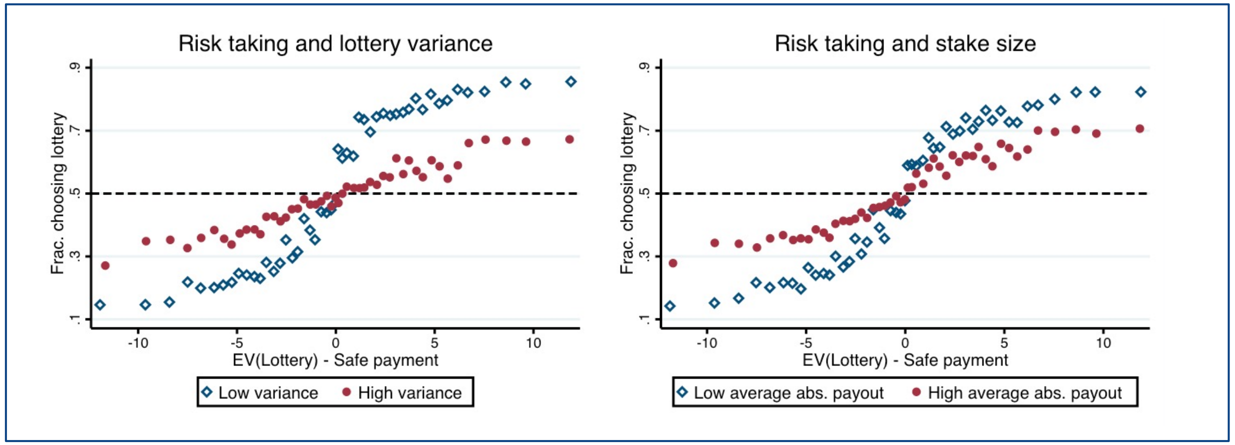


- Intuition: If a choice problem is super complex, DM may just randomly pick one option
- Group by OPC/SPC, split at median
  - Higher OPC/SPC choices are more compressed to 50/50 choice rate
- Group by EV, A or B higher
  - Noiseless DM would choose the one with higher EV 100% of time
  - Strong correlation between OPC and and fraction of “incorrect” choice $r = 0.66$
  - Controlled for EV difference as well
- Explanatory power
  - OPC >> EV difference
  - OPC >> estimated prospect theory (PT) value differences

Results III: Lottery vs Certain Payment



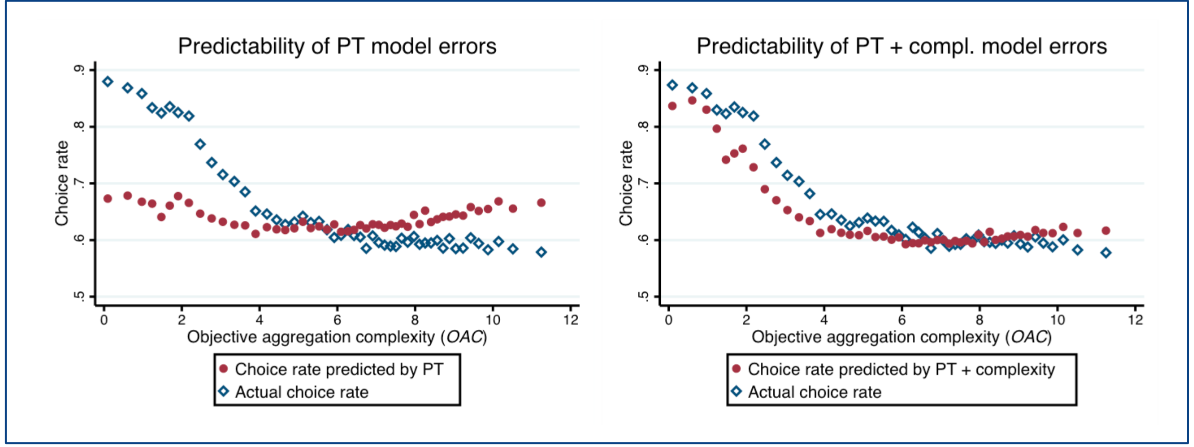
- Examined with safe payment
- Similar compression effect
- No complexity-aversion
  - Seeking on the left to 0
  - Averse on the right to 0
  - Complexity seems more of a problem-level feature taken as given by DM instead of an unfavorable option-level quality

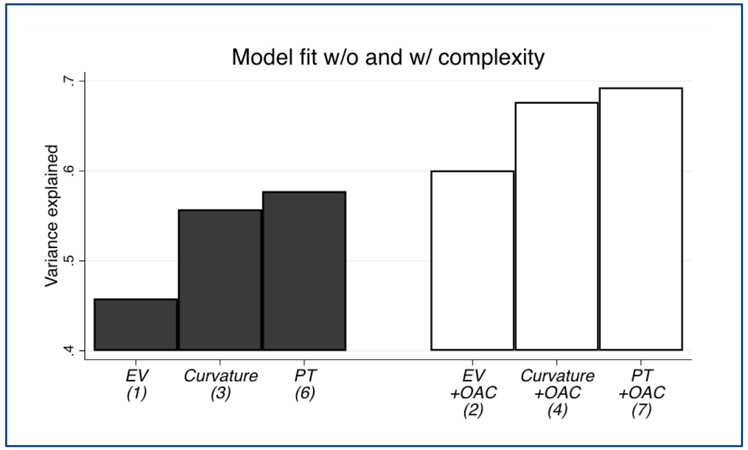


- Small-stake risk-aversion
  - Restricted to problems with weakly positive options and safe payment
  - Estimate CRRA expected utility model
    - $EU(x) = \mathbb{E}[x^{\alpha}]$
    - $\hat{\alpha} = 0.77$, risk-averse on the left to 0
    - $\hat{\alpha} = 1.04$, risk-loving on the right to 0
- Effect from variance (higher payout also means higher variance)
- Complexity-dependent noise/heteroskedasiticity

Results IV: Prediction of Choice

- Prediction of choice: $\mathrm{P(A)} = \frac{1}{1+e^{-\left(\eta_0+\eta_1\frac{1}{OAC_{A,B}}\right)[F(A)-F(B)]}}$
- Variations:
  - Objective function: EV, EU, or prospect theory
  - Complexity-dependent noise: $\eta_1 = 0$ or η_1 is estimated
- Results:
  - EV: expected value maximization and only estimates a constant error variance
  - Curvature: adds separate utility curvature parameters for gains and losses for reference-dependent adjustment
  - PT: adds loss aversion and two probability weighting parameters





Discussion

1. Strong assumption on relationship between EV task and choice task
    - Does error in calculating EV mirror error in assessing preferred lottery choice?
    - Computational-level difference: output, higher EV = preference?
    - Maybe the algorithmic process of choice task is also totally different from EV calculation i.e. state aggregation

2. Validity of complexity indices
    - Is CNN a proper benchmark for explanatory power?
    - Could there be other higher-level abstract indices/features that does not make intuitive sense to human and cannot easily be recognized but actually play an important role?

3. Extensions
    - By authors: real-world assets, larger choice set, continuous valuation tasks
    - Interactions with offloading, social learning, ambiguity, etc.
    - Analogy to intertemporal choice, etc.
    - Neural representation of (perceived/subjective) complexity?

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